

Heterogeneous Fusion with a combined Evidential, Probability and OWA Methods for Target Classification

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Abstract – *Fusing multiple independent sensor measurements and human intelligence reports are essential to support critical decisions in a timely manner for today’s situation awareness systems. The problem of great significance is associated with fusing Human Originated Information (HOI) with the information from other sources. Ordered Weighted Average (OWA) algorithm was proposed recently as a means to assimilate uncertain human originated information. We propose a novel method to use OWA in conjunction with Bayesian and Dempster-Shafer Theory (DST) fusion algorithms to fuse information from diverse sources and demonstrate its effectiveness using an example from literature.*

Keywords: Tracking, Heterogeneous Fusion, Maritime Interdiction, Fuzzy Logic, Bayesian, Dempster Shafer Theory, and OWA.

1 Introduction

In a typical heterogeneous information fusion scenario, information arrives from the sensors continuously in all kinds of systems. The output of these sensors could include verbal reports, e-mails, digital images, electronics state-space information, and other types of information that need to be fused into a coherent picture to enhance situation awareness, so that the best decision can be made based on all available data. A sample of a target engagement operational scenario is depicted in Figure 1, where the engagement command is based on the heterogeneous fusion. In order to accommodate the dynamic nature of multi-attribute information, the fusion system is expected to aggregate multi-format information within the databases, files, and/or on web pages. Our proposed heterogeneous fusion model assumes two networked sensors such as Electronics Support Measures (ESM) sensors working cooperatively, and the HOI is modeled as an additional sensor to enhance the fusion

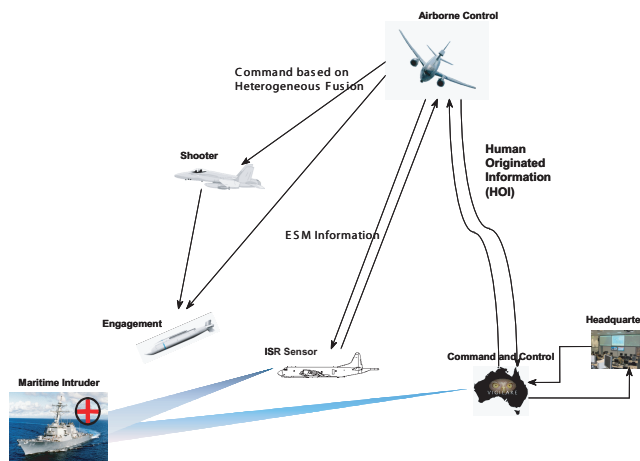


Figure 1: High Level Operational Graphics for Maritime Interdiction with Heterogeneous Fusion

process. Each sensor platform, including a human-in-the-loop observer, makes a sequence of reports. From each observation, the physical sensors would be able to extract a set of features associated with the target, but only a written or linguistic report will be provided by the human observer. A sample of the report is characterised by the following statement: “A highly agile maritime target was observed at a high risk location at noon today. The vessel seems to have low volume of carriage onboard, and it makes medium level of noise. Moderate level of response is provided by the target, and the shape of the vessel also matches one of the hostile ship types on our record with high volume of prior history. It seems like a High risk maritime target.” This kind of HOI is often a fuzzy mix of multiple criteria information, with highly agile, low volume, and medium noise embedded messages. In order to fuse this kind of in-

formation, the fusion algorithm will be required to deal with a mix of diverse qualitative/quantitative data with multiple levels of abstraction. Different computational technologies may be needed to handle the various stages of fusion, such that all information is fully exploited. Previous works based on transformation method were proposed in [8] to fuse probabilistic/evidential data for target identification problems. However, this method was comparatively restrictive to fuse information that is difficult to quantify, fuzzy, and vague such as HOI. This paper extends the concept in [8] to include a set of multi-criteria information based on the adaptation of an Ordered Weighted Averaging (OWA) approach [7]. In this method, the HOI will be first converted through a set of credibility values based on a number of transformation rules. The aggregated credibilities will then be re-scaled such that it is ranged monotonically between zero and unity. The method proposed in this paper provides an additional dimension to aggregate multi-criteria information that also leads to the insight of information compatibility, which is not widely addressed by the traditional fusion methods. The remainder of the paper is structured as follows: In Section 2, we introduce Bayesian, Dempster-Shafer Theory, and Ordered Weighted Average (OWA) methods. In Section 3 and Section 4, approaches will be given to adopt the theories discussed, and to demonstrate the techniques of fusing probabilistic, evidential and fuzzy linguistic information. Finally, Section 5 adopts an example from previous literature [9] to demonstrate the concept of heterogeneous fusion under a centralised and distributed construct. Section 6 will provide some concluding remarks to summarise this work, and provide suggestions for future research activities in this area.

2 Overview of Bayesian, Dempster-Shafer and OWA methods for Fusion

2.1 Bayesian Method

Let $\Omega = \{x_1, \dots, x_n\}$ denotes the set of possible target types and $\{S_1, \dots, S_n\}$ denotes the set of surveillance sensors in the network. For the sensor data, let $\{y_i^1\}$ denotes the current measurement from sensor S_i . Y_i^0 denotes the set of all previous measurements from sensor S_i . Y_i^1 denotes the set of all measurements from sensor S_i up to and including the current time. Then the task is to ascertain the target type x_j given to all available sensor measurements. In a standard Bayesian approach, the a posteriori probability of each target type given the totality of measurements is calculated recursively via Bayes theorem and then the Maximum A Posteriori (MAP) probability principle is used to declare a target type if a hard decision is required. Assuming the local data processing at node i , for target identification, employs the Bayesian approach, then for

each target type x_j :

$$\begin{aligned} p(x_j|Y_i^1) &= p(x_j|y_i^1, Y_i^0) \\ &= \frac{L(y_i^1|x_j, Y_i^0)p(x_j|Y_i^0)}{p(y_i^1|Y_i^0)} \\ &= C^{-1}L(y_i^1|x_j)p(x_j|Y_i^0) \end{aligned} \quad (1)$$

where $L(y_i^1|x_j) = p(y_i^1|Y_i^0)$ is the likelihood of observing measurement y_i^1 if the true target type is x_j , and $p(x_j|Y_j^0)$ is the a priori probability that the target type is x_j . Assuming the sensors are conditionally independent of each other given the target type, $L(y_i^1|x_j, Y_i^0) = L(y_i^1|x_j)$ and the term $C = p(x_j|Y_j^0)$ is constant with respect to x_j . As such, the a posteriori probabilities must be added to unity. Having updated the a posteriori probabilities in a single node scenario, the MAP principle may be invoked to declare the target type as x_k where, $k = \arg \max_{j=1 \dots k} p(x_j|Y_i^1)$. In the centralised fusion, the Bayesian approach is conducted in a similar way. Assuming the local data processing at node i , with the latest set of data: $p(x|Y_1^1 Y_1^2)$, and assuming the sensor measurements are independent, such that $p(y_1^1 y_1^2|x, Y_0^1 Y_0^2) = p(y_1^1|x, Y_0^1)p(y_1^2|x, Y_0^2)$, the Bayesian rule of fusion now becomes:

$$\begin{aligned} p(x|Y_1^1 Y_1^2) &= \frac{p(y_1^1|x, Y_0^1)p(y_1^2|x, Y_0^2)p(x|Y_0^1 Y_0^2)}{p(y_1^1 y_1^2|Y_0^1 Y_0^2)} \\ &= \frac{L(y_1^1|x, Y_0^1)L(y_1^2|x, Y_0^2)p(x|Y_0^1 Y_0^2)}{p(y_1^1 y_1^2|Y_0^1 Y_0^2)} \end{aligned} \quad (2)$$

Where $L(y_1^1|x, Y_0^1)$ is the likelihood for sensor Y_1^1 and $L(y_1^2|x, Y_0^2)$ is the likelihood for sensor Y_1^2 . Thus for the centralised fusion to operate, the principle idea would be the transformation of each sensor measures into a likelihood function.

2.2 Evidential Method and Belief Update with DST

In the DST approach [3], evidence about the target identity from each sensor measurement is represented in the form of a belief function $\langle F, m \rangle$ on the set of target frame of discernment Ω . With the new sensor measurement, the belief function will be updated based on the pooled new evidence. Assuming $\langle F_{y_i^1}, m_{y_i^1} \rangle$ is the belief function under the latest measurement of Y_i^1 , and $\langle F_{Y_i^0}, m_{Y_i^0} \rangle$ is the local a priori belief function based on all of the previous measurements. Then the local a posteriori belief function is as follow;

$$\langle F_{Y_i^1}, m_{Y_i^1} \rangle = \langle F_{y_i^1}, m_{y_i^1} \rangle \oplus \langle F_{Y_i^0}, m_{Y_i^0} \rangle \quad (3)$$

Using the Dempster's rule of combination, the new belief function is reproduced as follows. For each $A \subseteq \Omega$

$$(m_{y_i^1} \oplus m_{Y_i^0})(A) = \frac{1}{1 - K} \sum_{F \cap G = A} m_{y_i^1(F)}(F) m_{Y_i^0}(G) \quad (4)$$

where

$$K = \sum_{F \cap G = \emptyset} m_{y_i^1(F)}(F) m_{Y_i^0}(G) \quad (5)$$

The updated set of focal element is:

$$(F_{y_i^1} \oplus F_{Y_i^0}) = \{F \cap G | F \in F_{y_i^1}, G \in F_{Y_i^0}, F \cap G \neq \emptyset\} \quad (6)$$

Having calculated the posteriori belief function, the MAP principle may be applied to its pignistic probability distribution $\{BetProb_{Y_i^1}(x_j)\}_{j=1}^k$ to declare the target type as x_κ where,

$$\kappa = \arg \max_{j=1, \dots, k} BetProb_{Y_i^1}(x_j) \quad (7)$$

The concept of pignistic probability (BetP) mainly relies on the transformation of belief function $\langle F, m \rangle$ by setting,

$$BetProb(x_j) = \sum_{F \in \mathbf{F} | x_j \in F} \frac{m(F)}{|F|(1 - m(\emptyset))} \quad (8)$$

for each x_j in the frame of discernment Ω .

2.3 Ordered Weighted Average (OWA) Method

The concept of OWA operator was proposed in [5] to assist decision making in the presence of multi-criteria based linguistic information. Definition: A mapping F from

$$I^r \rightarrow I \text{ (where } I = [0,1]) \quad (9)$$

is called an OWA operator of dimension r if associated with F and W , is a weighting vector,

$$W = [w_1, w_2, \dots, w_r]^T \quad (10)$$

such that (1) $w_i \in (0, 1)$ and (2) $\sum_i w_i = 1$, where;

$$F(a_1, a_2, \dots, a_r) = w_1 b_1 + w_2 b_2 + \dots + w_r b_r \quad (11)$$

where b_1 is the i^{th} largest element in the collection a_1, a_2, \dots, a_r . Let's assume an r^{th} vector B , is an ordered argument vector, and each element $b_i \in [0, 1]$ and $b_i \geq b_j$ if $j > i$. Also assuming that an OWA operator F with weight vector W and an argument tuple (a_1, a_2, \dots, a_r) we can associate with this tuple an ordered input vector B such that B is the vector consisting of the arguments of F in descending order, and we get;

$$F(a_1, \dots, a_r) = W^T B \quad (12)$$

Sometimes it is convenient to denote $F(a_1, \dots, a_r)$ as $F(B)$ where B is the associate ordered argument vector. It should be noted that, the weight w_i is associated with a particular criteria position, rather than the criteria element. That means w_i is the weight associated with the i th largest element whichever component it is. Three most notable special cases of OWA aggregations are:

- *Max*: where $w^* = (1, \dots, 0)^T$ and $\max(a_1, \dots, a_r) = \max\{a_1, \dots, a_r\}$
- *Min*: where $w_* = (0, \dots, 1)^T$ and $\min(a_1, \dots, a_r) = \min\{a_1, \dots, a_r\}$
- *Average*: In this case $w_A = (1/r, \dots, 1/r)^T$ and $F_A(a_1, \dots, a_r) = \frac{a_1 + \dots + a_r}{r}$

Many Multi Criteria Decision Analysis (MCDA) techniques exist, and OWA is a specific class of technique to aggregate ordered criteria for a decision process. It's unique cumulative ordered outcome F was proved to be suitable to execute multiple criteria decision based on linguistic information. It is because this kind of information such as fuzzy HOI, is often seen as a type of aggregation function, where it is not exactly a pure 'ANDing' nor 'ORing' of multiple concerns. In many cases, the formulation desired lie somewhere between these two extremes. In the context of OWA method, this is adjusted by the variation of the weight w , vector. Compensative connectives have the property that a higher degree of satisfaction of one of the criteria can compensate for a lower degree of satisfaction of another criterion. *Oring* the criteria means full compensation and *Anding* the criteria means no compensation. The degree of *ORNESS* associated with the OWA operator is usually given as;

$$ORNESS(w) = \frac{1}{r-1} \sum_{i=1}^r (r-i)w_i \quad (13)$$

For any w the *ORNESS*(w) is always in unit interval. Also the nearer w is to an *OR*, the closer its measure is to one, while the nearer it is to an *AND*, the closer is to zero. It other words, in can be seen as the degree of optimism for a multi criteria decision.

2.3.1 Using Linguistic Quantifier Method to Obtain Weight

As proposed in [6], the standard degree of orness can be associated with a Regular Increasing Monotone (RIM) linguistic quantifier Q , which can provide information aggregation procedures guided by verbally expressed concepts.

$$ORNESS(Q) = \int_0^1 Q(h)dh \quad (14)$$

This definition of *ORNESS* quantifier measure provides a simple method of obtaining the OWA measures. Consider the family of RIM quantifiers

$$Q_\alpha(h) = h^\alpha, \quad \alpha > 0 \quad (15)$$

Then,

$$ORNESS(Q_\alpha) = \int_0^1 h^\alpha dh = \frac{1}{\alpha + 1} \quad (16)$$

and $ORNESS(Q_\alpha) < 0.5$ for $\alpha > 1$, $ORNESS(Q_\alpha) = 0.5$ for $\alpha = 1$, and $ORNESS(Q_\alpha) > 0.5$ for $\alpha < 1$. In [5], Yager suggested an approach to the aggregation of criteria satisfactions guided by a regular non-decreasing quantifier Q . If Q is RIM quantifier then we measure the overall success of the alternative $x = (a_1, \dots, a_r)$ by;

$$F_Q(a_1, \dots, a_r)$$

where F_Q is an OWA operator derived from Q , i.e. the weights associated with this quantifier-guided aggregation are obtained as follows

$$w_i = Q\left(\frac{i}{r}\right) - Q\left(\frac{i-1}{r}\right) \quad (17)$$

for $i = 1, \dots, r$.

2.3.2 Using Analytic Approach to Obtain Weights

The determination of weights is a very important step to establish the appropriate level of ORNESS, when applying OWA operator for decision making. The approach suggested in Section 2.3.1 summarises the use of linguistic quantifier method to determine the appropriate weights. However, this approach is very subjective, and it is difficult to quantify the optimum weight distribution across the OWA operator. In conjunction with the introduction of ORNESS, Yager also introduced the measure of dispersion for OWA operator. This measure is defined as:

$$disp(W) = - \sum_{i=1}^n w_i \ln w_i \quad (18)$$

This expression measures the degree to which W takes into account all information in the aggregation. O'Hagan [10] proposed the first analytical approach to address a special class of OWA operators weights using the dispersion measures. A recent literature survey [11] summarised a number of similar attempts published in the public domain. The heterogeneous fusion method in this paper adopted the analytical approach based on the maximal entropy OWA operator weight algorithm in [12], where the weight vector can be obtained by;

$$\ln w_j = \frac{j-1}{n-1} \ln w_n + \frac{n-j}{n-1} \ln w_1 \quad (19)$$

$$w_j = \sqrt[n-1]{w_1^{n-j} w_n^{j-1}} \quad (20)$$

and

$$w_n = \frac{((n-1)\alpha - n)w_1 + 1}{(n-1)\alpha + 1 - nw_1} \quad (21)$$

then

$$w_1[(n-1)\alpha + 1 - nw_1]^n = ((n-1)\alpha)^{n-1} [((n-1)\alpha - n)w_1 + 1] \quad (22)$$

where α is the degree of ORNESS and n is the number of criterion under consideration. Details of the analytical proof are outlined in the Fuller and Majlender paper [12]. This concept of using OWA as part of the fusion will be further explored with an example illustrated in the sections below.

3 Modelling the Human Originated Information

In order to incorporate the linguistic information, the OWA method adopted the four main steps approach in [13] with moderate tailoring to accommodate the heterogeneous fusion. The first step collects a set of credibility components based on the linguistic input:

$$A_k = \{a_1, \dots, a_n\} \quad (23)$$

A is the summary of a HOI report at time k , and n is the number of credibility components extracted from the HOI report. Ideally, the HOI report consists of a set of descriptive language for a scene, object, or movement under observation, but it can also be summarised by a set of the credibilities A_k . With the assumption that all HOI reports consist of a full set of criteria, the second step is to convert the A_k to a set of target specific credibility value, $a_{r_{Ti}}$, where Ti is the target type number. The credibility translation is primarily conducted through a look-up table, where specific characteristics were pre-assigned for each target type. The look-up table is basically a knowledge-base prepared by a group of Subject Matter Experts (SME). In the context of this modelling, we called this knowledge-base as Criteria Signature, which is similar to Table 1. Third component in the proposed approach is to introduce a series of expert opinions, that are solicited in ranking the alternatives. The ranking system is a two stage process. In the first stage, the expert is asked to provide an evaluation of the alternatives. This evaluation consists of a rating for each alternative on each of the criteria, where the ratings are chosen from the scale $\{1, 2, 3\}$. The next step in the process is to find the overall evaluation for an alternative by an assessment for each target type. Upon receiving the HOI report, it is first summarised into a set of criteria similar to Table 2 before a matching process is conducted. The characteristics of seven soft-attributes considered in this classification include, *Risk of Sight Location* - a_1 , *Volume of Carriage* - a_2 , *Volume of Prior History* - a_3 , *Responsiveness* - a_4 , *Agility* - a_5 , *Noise Level* - a_6 , and *Risk Assessment* - a_7 . For an illustrative purpose, each target type only uses three distinct criteria for their classifier. A summary of the classifier conversions are listed in Tables 3 to 6. Although the proposed method has a potential to simultaneously collate a number of HOI reports, this example will only explore the option of one HOI report at a time to simplify the problem. This modelling work will examine four target types, and

Criteria a_{T_i}	T1	T2	T3	T4
Sight Location Risk - a_1	High	Low	Low	High
Volume of Carriage - a_2	Low	High	Low	Low
Volume of Prior History - a_3	High	Low	Low	Low
Responsiveness - a_4	Low	High	High	Low
Agility - a_5	Low	High	Low	High
Noise Level - a_6	High	Low	High	High
Risk Assessment - a_7	High	Low	High	Low

Table 1: HOI Criteria Signature

Criteria	A_{SL}	A_C	A_H	A_R	A_A	A_N	A_{RISK}
Report(a)	high	low	high	medium	low	medium	high

Table 2: HOI Criteria Summary

each target possesses a set of hard-attributes that can be identified by physical sensor based on the partial probability databases in Figure 2. In this work, credibility characterisation is performed on each target type T_i , the aggregated Credibility for each target type is calculated based on the OWA method of;

$$F(a_1, \dots, a_r)_{T_i} = \sum_{r=1}^n w_r b_r \quad (24)$$

where b_r is the r^{th} largest element of $\langle a_1, \dots, a_r \rangle$, and i is the type number of the target. w_r is the weight associated with the OWA operator, which is calculated by either a linguistic quantifier method or analytical method described above. The case study illustrated in this paper use both techniques, and the results will be compared as part of the conclusion.

3.1 Linguistic Quantifier Method for OWA Weights

Taking into consideration that, there are four possible target names label as T_i , and each target has 7 criteria, the weights derived from Q_α are determined based on Equation (17) where $i = 7$. Furthermore, whatever is the linguistic quantifier, Q_α , representing the statement *most criteria are satisfied*, we see that

$$1 \leq F_\alpha(a_1, a_2, a_3, a_4, a_5, a_6, a_7) \leq 3$$

holds for each alternative $A_k = \mathbf{a} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ since $a_r \in \{1, 2, 3\}$. In order to search for an index $\alpha \geq 0$, such that the linguistic quantifier Q_α is as close to the experts' opinion on each target type as possible. A statement of boundary calibration for each target type is defined by the expert as follows:

Criteria a_{T_i}	T1		
	High	Medium	Low
Sight Location Risk(SL)	$a_{T1,SL} = 3$	$a_{T1,SL} = 2$	$a_{T1,SL} = 1$
Volume of Carriage(C)	$a_{T1,C} = 1$	$a_{T1,C} = 2$	$a_{T1,C} = 3$
Prior History Volume(H)	$a_{T1,H} = 3$	$a_{T1,H} = 2$	$a_{T1,H} = 1$
Responsiveness(R)	$a_{T1,R} = 1$	$a_{T1,R} = 2$	$a_{T1,R} = 3$
Agility(A)	$a_{T1,A} = 1$	$a_{T1,A} = 2$	$a_{T1,A} = 3$
Noise Level(N)	$a_{T1,N} = 3$	$a_{T1,N} = 2$	$a_{T1,N} = 1$
Risk Assessment(Risk)	$a_{T1,Risk} = 3$	$a_{T1,Risk} = 2$	$a_{T1,Risk} = 1$

Table 3: HOI Criteria Summary for Type 1 target

Criteria a_{T_i}	T2		
	High	Medium	Low
Sight Location Risk(SL)	$a_{T2,SL} = 1$	$a_{T2,SL} = 2$	$a_{T2,SL} = 3$
Volume of Carriage(C)	$a_{T2,C} = 3$	$a_{T2,C} = 2$	$a_{T2,C} = 1$
Prior History Volume(H)	$a_{T2,H} = 1$	$a_{T2,H} = 2$	$a_{T2,H} = 3$
Responsiveness(R)	$a_{T2,R} = 3$	$a_{T2,R} = 2$	$a_{T2,R} = 1$
Agility(A)	$a_{T2,A} = 3$	$a_{T2,A} = 2$	$a_{T2,A} = 1$
Noise Level(N)	$a_{T2,N} = 3$	$a_{T2,N} = 2$	$a_{T2,N} = 1$
Risk Assessment(Risk)	$a_{T2,Risk} = 1$	$a_{T2,Risk} = 2$	$a_{T2,Risk} = 3$

Table 4: HOI Criteria Summary for Type 2 target

Criteria a_{T_i}	T3		
	High	Medium	Low
Sight Location Risk(SL)	$a_{T3,SL} = 1$	$a_{T3,SL} = 2$	$a_{T3,SL} = 3$
Volume of Carriage(C)	$a_{T3,C} = 1$	$a_{T3,C} = 2$	$a_{T3,C} = 3$
Prior History Volume(H)	$a_{T3,H} = 1$	$a_{T3,H} = 2$	$a_{T3,H} = 3$
Responsiveness(R)	$a_{T3,R} = 3$	$a_{T3,R} = 2$	$a_{T3,R} = 1$
Agility(A)	$a_{T3,A} = 1$	$a_{T3,A} = 2$	$a_{T3,A} = 3$
Noise Level(N)	$a_{T3,N} = 3$	$a_{T3,N} = 2$	$a_{T3,N} = 1$
Risk Assessment(Risk)	$a_{T3,Risk} = 3$	$a_{T3,Risk} = 2$	$a_{T3,Risk} = 1$

Table 5: HOI Criteria Summary for Type 3 target

1. Type 1 Target

- If Type 1 target has more than 4 Low criteria, then it's overall credibility should be less than 2.
- If Type 1 target has more than 2 high criteria, then it's overall credibility should be greater than 2.75.

2. Type 2 Target

- If Type 2 target has more than 2 weak criteria, then it's overall credibility should be less than 1.8.
- If Type 2 target has more than 4 high criteria, then it's overall credibility should be greater than 2.75.

3. Type 3 Target

- If Type 3 target has more than 5 weak criteria, then it's overall credibility should be less than 1.8.
- If Type 3 target has more than 2 high criteria, then it's overall credibility should be greater than 2.75.

4. Type 4 Target

- If Type 4 target has more than 3 weak criteria, then it's overall credibility should be less than 1.5.

Criteria a_{T_i}	T4		
	High	Medium	Low
Sight Location Risk(SL)	$a_{T4,SL} = 3$	$a_{T4,SL} = 2$	$a_{T4,SL} = 1$
Volume of Carriage(C)	$a_{T4,C} = 1$	$a_{T4,C} = 2$	$a_{T4,C} = 3$
Prior History Volume(H)	$a_{T4,H} = 1$	$a_{T4,H} = 2$	$a_{T4,H} = 3$
Responsiveness(R)	$a_{T4,R} = 1$	$a_{T4,R} = 2$	$a_{T4,R} = 3$
Agility(A)	$a_{T4,A} = 3$	$a_{T4,A} = 2$	$a_{T4,A} = 1$
Noise Level(N)	$a_{T4,N} = 3$	$a_{T4,N} = 2$	$a_{T4,N} = 1$
Risk Assessment(Risk)	$a_{T4,Risk} = 1$	$a_{T4,Risk} = 2$	$a_{T4,Risk} = 3$

Table 6: HOI Criteria Summary for Type 4 target

- If Type 5 target has more than 4 high criteria, then it's overall credibility should be greater than 2.5.

For the first agreement for target Type 1 we get

$$F_{\alpha_{T_1}}(3, 3, 1, 1, 1, 1, 1) = 3 \times (w_1 + w_2) + w_3 + w_4 + w_5 + w_6 + w_7 < 2$$

that is,

$$3 \times \left[\frac{2}{7}\right]^\alpha - \left\{ \left[1 - \frac{1}{3}\right]^\alpha \right\} < 2 \iff \alpha_{T_1} < 0.55$$

For the second agreement for target Type 1 we also get

$$F_{\alpha_{T_1}}(3, 3, 3, 1, 1, 1, 1) = 3 \times (w_1 + w_2 + w_3) + w_4 + w_5 + w_6 + w_7 > 2.75$$

that is,

$$3 \times \left[\frac{3}{7}\right]^\alpha + 1 - \left[\frac{3}{7}\right]^\alpha > 2.75 \iff \alpha_{T_1} > 0.158$$

based on the agreement from Type 1 statement, α_{T_1} should be set between the following;

$$0.16 < \alpha_{T_1} < 0.55$$

In a similar way, the degree of quantifier for Type 2, 3 and 4 targets are calculated below;

$$0.39 < \alpha_{T_2} < 1.64$$

$$0.158 < \alpha_{T_3} < 0.47$$

$$0.86 < \alpha_{T_3} < 1.636$$

To illustrate the concept, we take an examination to investigate the HOI report in Section 1. The summary of the HOI report is outlined in Table 2, where the conversion in Tables 3 to 6 are adopted for the following simulation model.

$$B(a_1, a_2, a_3, a_4, a_5, a_6, a_7)_{T_1} = \{3, 3, 3, 2, 3, 2, 3\}$$

$$B(a_1, a_2, a_3, a_4, a_5, a_6, a_7)_{T_2} = \{1, 1, 1, 2, 1, 2, 1\}$$

$$B(a_1, a_2, a_3, a_4, a_5, a_6, a_7)_{T_3} = \{1, 3, 1, 2, 3, 2, 3\}$$

$$B(a_1, a_2, a_3, a_4, a_5, a_6, a_7)_{T_4} = \{3, 3, 1, 2, 1, 2, 1\}$$

Taking the consideration of previous estimation on α , the values on each linguistic quantifier interval α are chosen as $\alpha_{T_1} = 0.5$, $\alpha_{T_2} = 1.2$, $\alpha_{T_3} = 0.4$, and $\alpha_{T_4} = 1$. In this way, the weight for each target classifier becomes the following values;

$$W_{T_1} = (w_1, w_2, w_3, w_4, w_5, w_6, w_7) = \{0.38, 0.16, 0.12, 0.10, 0.09, 0.08, 0.07\}$$

and similarly $W_{T_2} = \{0.09, 0.13, 0.14, 0.15, 0.16, 0.16, 0.17\}$, $W_{T_3} =$

$\{0.46, 0.15, 0.11, 0.09, 0.07, 0.07, 0.06\}$, and $W_{T_4} = \{0.14, 0.14, 0.14, 0.14, 0.14, 0.14, 0.14\}$. Based on the weight vector, the aggregated OWA credibility for each classifier becomes:

$$F_{T_1} = W_{T_1} B_{T_1} = 2.85$$

Also, $F_{T_2} = W_{T_2} B_{T_2} = 1.45$, $F_{T_3} = W_{T_3} B_{T_3} = 2.43$, and $F_{T_4} = W_{T_4} B_{T_4} = 2.1$. Finally a normalisation process is conducted to the aggregates, and producing $\bar{F}_{T_1} = 0.32$, $\bar{F}_{T_2} = 0.17$, $\bar{F}_{T_3} = 0.27$, and $\bar{F}_{T_4} = 0.28$. Based on this calculation, we claim that there is a 32% chance that we have identified a Type 1 target at time k_1 , but we also have a 28% chance that it is a Type 4 target. The authors would note that, this identification methodology is purely relied on descriptive information outlined in Section 3.1, where the identification process is conducted through human-in-the-loop system. Although the \bar{F}_{T_i} is an aggregated criterion assembled by the HOI, it is also being treated as a likelihood here for target classification.

3.2 Analytical Approach for OWA Weights

Prior to the classification process to take place, an analytical method outlined in Section 2.3.2 is applied to determine a set of optimum weights for the aggregates. The α value discussed by Fuller and Majlender [12] reflect the overall ORNESS or optimism associate with the aggregation conclusion. In that way, we could convert the HOI descriptions to the most appropriate likelihood based on the optimism to each classifier. Suppose the optimism of our classifiers is set to 0.6, where 1 is the most optimistic, and 0 is the most pessimistic. α is subsequently set to 0.6 for this heterogeneous fusion example. With $n = 7$ of criteria, $\alpha = 0.6$, and using Equation (22), we find;

$$\begin{aligned} w_1^* &= 0.216 \\ w_7^* &= \frac{[(n-1)\alpha - n]w_1 + 1}{(n-1)\alpha + 1 - nw_1} \\ &= \frac{[(7-1)\alpha - 7]0.216 + 1}{(7-1)\alpha + 1 - 7 \times 0.216} \\ &= 0.3388 \\ w_j^* &= \sqrt[n-1]{w_1^{n-j} w_n^{j-1}} \\ w_2^* &= \sqrt[7-1]{w_1^{7-2} w_7^{2-1}} = 0.2328 \end{aligned}$$

and similarly, $w_3^* = 0.2510$, $w_4^* = 0.2705$, $w_5^* = 0.2916$, and $w_6^* = 0.3143$. As a result, the OWA operator with maximum dispersion with the ORNESS of $\alpha = 0.6$ become;

$$W^*(\alpha) = W^*(0.6) = [0.216, 0.2328, 0.2510, 0.2705, 0.2916, 0.3143, 0.3388]$$

Based on the analytical weight vector, the aggregated OWA credibility for each classifier becomes:

$$\begin{aligned}
 F_{T_1} &= W^* B_{T_1} \\
 &= [0.216, 0.2328, 0.2510, 0.2705, 0.2916, 0.3143, 0.3388] \\
 &\quad \times [3, 3, 3, 3, 3, 2, 2]^T \\
 &= 5.0915
 \end{aligned}$$

with the similar computational method, $F_{T_2} = 2.3638$, $F_{T_3} = 3.8767$, and $F_{T_4} = 3.3341$. Base on the process discussed in the previous section, a normalisation step is performed to the aggregates and become: $\bar{F}_{T_1} = 0.35$, $\bar{F}_{T_2} = 0.16$, $\bar{F}_{T_3} = 0.26$, and $\bar{F}_{T_4} = 0.23$. Following with the previous discussion, these normalised aggregates are also treated as a likelihood here for target classification. To demonstrate the concept, and to align with Blackman’s simulation to be described in the next section, we set Type 1 target as the truth in both OWA weight determination methods, and then execute a Monte Carlo simulation of 150 descriptions. Two set of simulations with two types of OWA were calculated. One data set consists of a simulated good HOI reports, and the other one with poor HOI reports. With the good reports, an average of 50% target Type 1’s criteria are recorded, but a roughly 20% of target Type 1’s criteria will be recorded by the poor descriptive report. Upon completion, these values are being fused as part of a centralised/distributed multi-sensors fusion process discussed previously.

3.3 Transforming HOI into Bayes’ Likelihood

The concept discussed in this paper first characterise each target type with a set of unique criteria signature, which is similar to the example illustrated in Table 1. In order to produce the likelihood function, the HOI is then compared with each criteria signature. This comparison allows a likelihood function to be produced based on the multiple criteria of each target under HOI description. Considering the proposed method is treating $F(a_1, \dots, a_r)_{T_i}$ as a likelihood of target detection, a normalised value will be calculated for each target before fusing with external heterogeneous information sources. This novel technique essentially provides a direct pathway to covert the fuzzy HOI into a computable likelihood function based on a set of unique criteria signatures. Our proposed heterogeneous fusion model assumes that one sensor collects information through HOI, and the other two sensors behave like two networked sensors working cooperatively to identify targets similar to the example articulated in [9]. Each platform possesses an on-board sensor S_i which makes a sequence of observations. From each observation, the on-board sensor would be able to extract a single feature f_i of the target and represent it as either a prob-

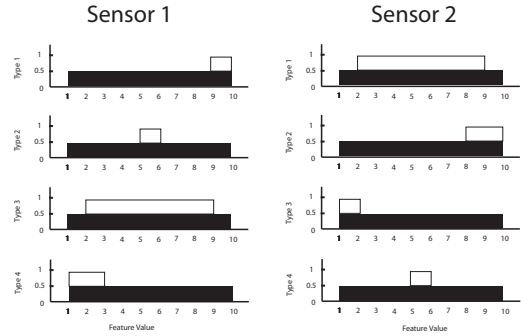


Figure 2: Partial Probability Databases

ability distribution or a belief function. At the same time, a HOI report similar to the one that was outlined in Section 1, which consist of soft-attributes such as prior history of the target, behavior, shape etc, are reported by an experienced human observer to describe the target.

4 Modelling Sensor Originated Information

The sensor platform adopted in this heterogeneous fusion paradigm is similar to the Radar Warning Receiver (RWR) sensors commonly used by air defence self-protection systems. The data synthesis method for the RWR sensor was largely covered in [8]. However, the summary is repeated in this section for the purpose of completeness. The values that each feature collected by this sensor can vary from target to target based on their operating mode. In the scenario discussed in this report, the classification sensor is being treated as a camera, which has the capability to produce hard-attribute. This type of sensor is commonly used on the Maritime ISR platform such as the one shown in Figure 1. Likewise, the values of its emitter characteristics will vary according to the emitter’s mode of operation. Finally, the feature extracted from the sensor can only be possible to determine the range in which the feature values will lie, but without knowing specifically which values in the range will be assumed. The example presented here assumes four targets are represented in terms of partial probabilities. This assumption is in parallel with the four targets illustrated in Section 3. The partial probability database for feature f_1 and f_2 are illustrated in Figure 2, where the horizontal axis represent the feature values. For each type, the black and white bars indicate possible modes that a target of such type may operate in. The vertical heights of the bars indicate the probabilities of those modes being used. As an example, for sensor 1, it can be seen that the probability of a target of type 3 operating in the ‘white’ mode is the same as the probability of it operating in the ‘black’ mode. If the ‘black’ mode is selected, then feature f_1 may assume values between 1 and 10,

while if the ‘white’ mode is selected, then it may only assume values between 5 and 6. For a given target type and feature, the process of observing such target by automated sensor is a two step process. First, the target’s feature value is ‘extracted’ in two stages. The first stage involves randomly selecting the target mode. This is done by generating a random number on the interval $[0, 1]$ and rounding it to the closet integer. If 0, then the ‘white’ mode is selected, otherwise the ‘black’ mode is selected. For the second stage, the feature range for that mode and target type is modeled by a uniform probability distribution defined on the same interval as the feature range. The feature value is then selected by randomly sampling from the distribution. Based on the principle discussed, we could generate the synthetic data by the automated sensors that are consistent with the feature model. The first technique known as the Power Set Method is used to generate belief functions defined on $\Omega = \{x_1, x_2, x_3, x_4\}$. The belief functions for features f_1 and f_2 have already been listed in Table 1 and Table 2 of [8], which will not be repeated in here. The second technique involves determining the likelihood of extracting the feature value f_i given each target type x_j according to equation (25).

$$L(f_i|x_j) = L(f_i|x_j, \text{target mode is white}) \cdot p(\text{target mode is white}) + L(f_i|x_j, \text{target mode is black}) \cdot p(\text{target mode is black}) \quad (25)$$

where $L(f_i|x_j, \text{target mode is white})$ and $L(f_i|x_j, \text{target mode is black})$ represent the uniform probability densities of feature f_i for the black and white target modes corresponding to target type x_j , and the probability $p(\text{target mode is white})$ and $p(\text{target mode is black})$ both equal one half. The likelihoods for features f_1 and f_2 are listed in Table 7 and Table 8 as a function of the feature values. The overall process of observing the target, extracting the feature value, and representing it as a set of likelihoods or belief function by the automated sensors was emulated by combining the two steps. At each instant, the mode and feature value were randomly selected by the given target type x_j and feature f_i , then the corresponding belief function was read from Table i and the corresponding likelihoods were read from Tables $i + 1$. For example, given target type x_2 and feature f_1 , suppose the ‘white’ mode was randomly selected and that the feature value 5.7 was generated, then the belief function determined by the basic belief

Feature range	$L(f_1 x_1)$	$L(f_1 x_2)$	$L(f_1 x_3)$	$L(f_1 x_4)$	Normalisation Constant
[1, 2]	11/36	1/18	1/18	1/18	17/36
[2, 3]	11/36	8/63	1/18	1/18	137/252
[3, 5]	1/18	8/63	1/18	1/18	37/126
[5, 6]	1/18	8/63	5/9	1/18	50/63
[6, 9]	1/18	8/63	1/18	1/18	37/126
[9]	1/18	8/63	1/18	5/9	50/63
[9, 10]	1/18	1/18	1/18	5/9	13/18

Table 7: Likelihoods for Sensor 1

Feature range	$L(f_1 x_1)$	$L(f_1 x_2)$	$L(f_1 x_3)$	$L(f_1 x_4)$	Normalisation Constant
[1, 2]	1/18	5/9	1/18	1/18	13/18
[2]	1/18	5/9	1/18	8/63	50/63
[2, 5]	1/18	1/18	1/18	8/63	37/126
[5, 6]	5/9	1/18	1/18	8/63	50/63
[6, 8]	1/18	1/18	1/18	8/63	37/126
[8, 9]	1/18	1/18	11/36	8/63	137/252
[9, 10]	1/18	1/18	11/36	1/18	17/36

Table 8: Likelihoods for Sensor 2

assignment are;

$$\begin{aligned} m(\{x_2, x_3\}) &= 6/17 \\ m(\{x_2, x_3, x_4\}) &= 4/17 \\ m(\{x_1, x_2, x_3\}) &= 4/17 \\ m(\{x_1, x_2, x_3, x_4\}) &= 3/17 \end{aligned}$$

would be read from Table 1 in [8], and used as the measurement for the subsequent fusion processing if the local processing of sensor 1’s output employed evidential reasoning, otherwise the likelihoods

$$\begin{aligned} L(f_1|x_1) &= 1/18 \\ L(f_1|x_2) &= 8/63 \\ L(f_1|x_3) &= 5/9 \\ L(f_1|x_4) &= 1/18 \end{aligned}$$

would be read from Table 7 and used as the measurement for the subsequent fusion processing if the local processing of sensor 1’s output employed Bayesian reasoning.

5 Heterogeneous Fusion Example

The algorithm presented here is based on Distributed Bayesian fusion which was previously presented in [8], but modification is introduced to incorporate the HOI. The algorithm initialises the value of each of the a priori probabilities $p(x_j|Y_1^0, \dots, Y_n^0)$ for each target type $j = 1, \dots, k$ at the central processing node in the network. Then proceeds to Step 1 if an observation is made at one of the sensor nodes. The centralised algorithm is processed in a similar way, but requires minor tuning. STEP 1: If the observation is made at sensor node i_1 and sensor node i_1 expresses the evidence from the measurement $y_{i_1}^1$ as a belief function $\langle F_{y_{i_1}^1}, m_{y_{i_1}^1} \rangle$, transform the belief function to a measurement probability distribution using the pignistic

probability transformation. STEP 2: If the observation is made at sensor node i_1 and sensor node i_1 expresses in meta data but collated in the form of HOI format. Transform the HOI data to an aggregated credibility $F(a_1, \dots, a_r)_{T_i}$ using the OWA techniques discussed above. The normalised $\bar{F}(a_1, \dots, a_r)_{T_i}$ will now become the measurement $y_{i_1}^1$. STEP 3: Transmit the probability distribution determined by the measurement $y_{i_1}^1$ to the central processing node. STEP 4: Interpreting the probability distribution as the set of likelihoods $\{p(y_{i_1}^1|x_j)\}_{j=1}^k$, use Bayes' probability updating equation to calculate the a posteriori probabilities $p(x_j|Y_{i_1}^1, Y_{i_2}^0, \dots, Y_{i_n}^0)$ for each $j = 1, \dots, k$. STEP 5: Communicate the global a posteriori probability distribution $\{p(x_j|Y_{i_1}^0, \dots, Y_{i_n}^0)\}_{j=1}^k$ to each of the other nodes. STEP 6: Reset each of the a priori probabilities $p(x_j|Y_{i_1}^0, Y_{i_2}^0, \dots, Y_{i_n}^0)$ to $p(x_j|Y_{i_1}^1, Y_{i_2}^0, \dots, Y_{i_n}^0)$ for all $j = 1, \dots, k$ ready for the next iteration. STEP 7: If a new observation is made at one of the sensor nodes, return to Step 1 and repeat the process. For comparison, a scenario assuming Type 1 target is within the vicinity of detection range by all three sensors, namely Probabilistic, Evidential and HOI sensors. A Monte Carlo simulation was employed to estimate the probability of correct target type identification. The detections are calculated by averaging the a posteriori probabilities, or pignistic probability, or aggregated Credibility for the type 1 target of hundred runs. In this simulation, each sensor made a hundred observations of the target during a single run for both centralised and distributed fusion. A non-informative priors (ie. Uniform probability distributions or vacuous belief functions) is used to initialise the algorithms in the scenarios. The results of fusing two sensors and three sensors based on the centralised heterogeneous information fusion are illustrated in Figure 3, where both poor/good HOI were used. Figure 3 demonstrates that, with the aid of an additional synchronised OWA based HOI sensor, the rate of detection for heterogeneous fusion was accelerated. A similar result is also achieved when the OWA operator weight is calculated through an analytical method as per outline in Figure 5 However, poor HOI reports are obviously not as helpful as a set of good reports, which result in slower convergence. Figure 4 and Figure 6 confirmed that the OWA based heterogeneous fusion has a similar enhancement capability to distributed fusion in a similar manner as the centralised fusion. Close examination to Figure 4 and Figure 6 suggested that, the detection performance assist by OWA based heterogeneous fusion method is not as pronounce as the centralised configuration. Given HOI reports are usually provided in a less frequent manner than the active sensors, a subsequent simulation was conducted to examine how extrapolated HOI reports could impact the overall heterogeneous fusion. To do that, despite a similar Monte Carlo simulation was employed for the HOI sensor, each report is now shared by at least 10 iterations of

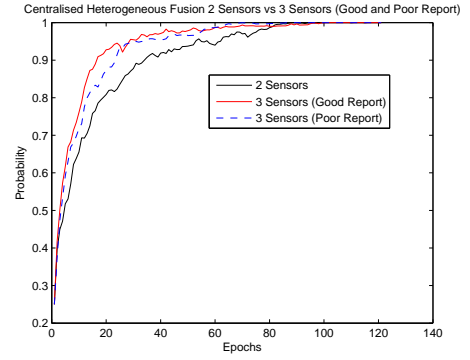


Figure 3: Centralised Heterogeneous Fusion with Synchronise HOI Reports - Linguistic Quantifier Method

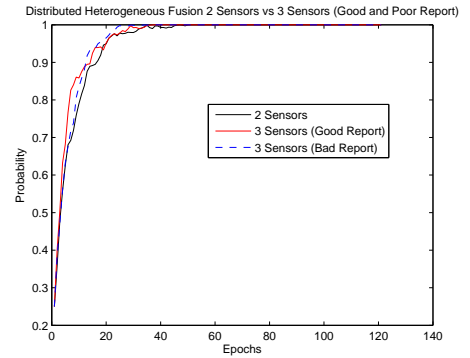


Figure 4: Distributed Heterogeneous Fusion with Synchronise HOI Reports - Linguistic Quantifier Method

Bayesian/Evidential fusion. Assuming Type 1 target is still our desire detection, results using the extrapolated HOI report for Centralised and Distributed fusion are illustrated in Figure 7 and Figure 8 respectively. A similar simulation was also conducted for analytical OWA operator weight, and the results are depicted in Figure 9 and Figure 10. Both simulations suggested that, extrapolated HOI reports do not induce a significant improvement to the overall effectiveness of heterogeneous fusion. However, given a situation when only one high quality HOI Report is received by the fusion centre, the simulation results illustrate in Figure 11 and Figure 12 confirmed that a reliable linguistic report could significantly enhance the Bayesian based fusion technique.

6 Concluding Remarks

Heterogeneous fusion has become a widely contested problem in the information fusion community. The technique of using OWA based method to combine with traditional probabilities/evidential based fusions was explored in this paper. The proposed heterogeneous method utilised both centralised and distributed fusion construct. The feature method highlighted a novel approach to incorporate a set of linguistic/descriptive in-

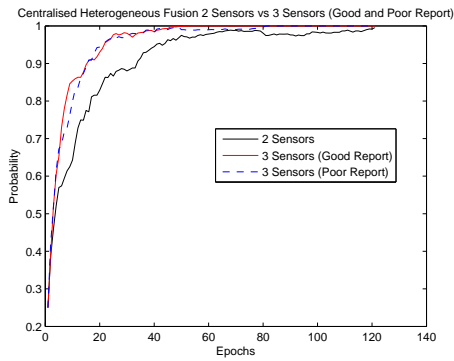


Figure 5: Centralised Heterogeneous Fusion with Syn-
chronise HOI Reports - Analytic Method

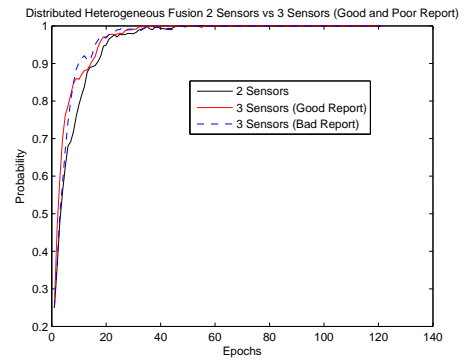


Figure 8: Distributed Heterogeneous Fusion with Ex-
trapolated HOI Reports - Linguistic Quantifier Method

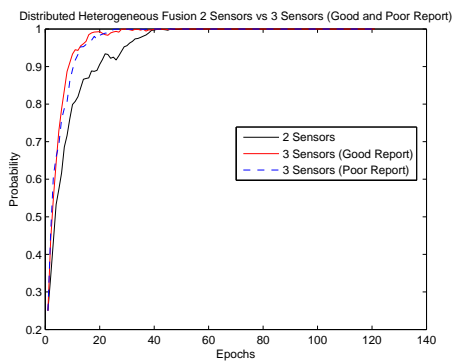


Figure 6: Distributed Heterogeneous Fusion with Syn-
chronise HOI Reports - Analytic Method

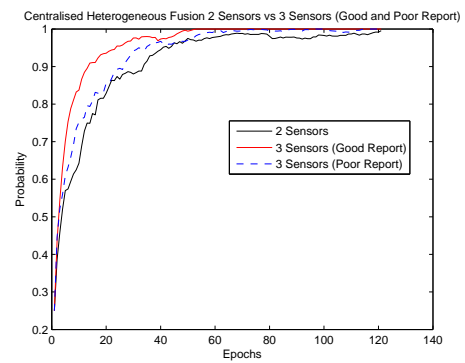


Figure 9: Centralised Heterogeneous Fusion with Ex-
trapolated HOI Reports - Analytic Method

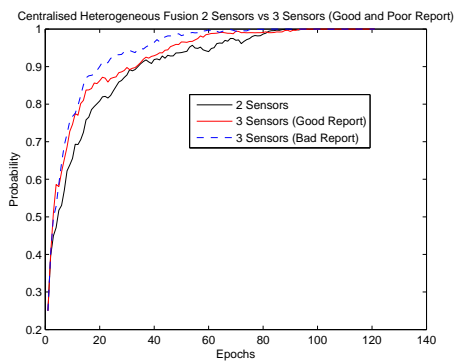


Figure 7: Centralised Heterogeneous Fusion with Ex-
trapolated HOI Reports - Linguistic Quantifier Method

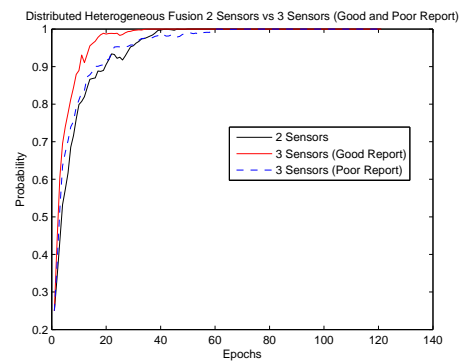


Figure 10: Distributed Heterogeneous Fusion with
Extrapolated HOI Reports - Analytic Method

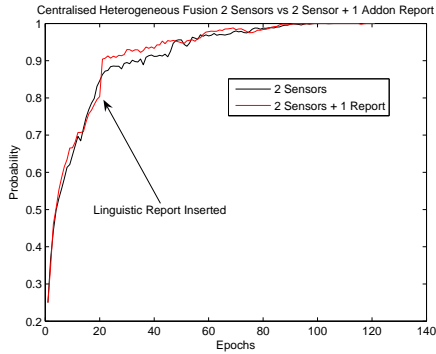


Figure 11: Centralised Heterogeneous Fusion with a Reliable HOI Report Insertion

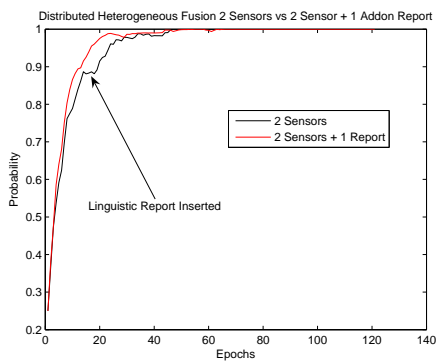


Figure 12: Distributed Heterogeneous Fusion with a Reliable HOI Report Insertion

puts, and transform the information into a set of fusible aggregated values. This proposed method has been tested on a small problem drawn from the literature with encouraging results. While the employed technique demonstrates a promising framework to fuse heterogeneous information, further works are required to examine a way to quantify the performance of this approach. For the purpose of articulating the heterogeneous fusion concept, the weight was assigned based on the basic method articulated in Yager’s original paper. However, additional work is needed to explore the implication of various OWA weight assigning methods discussed in [11], because the determination of OWA operator weight also governs the level of ‘orness’ - optimism, in any multi-criteria decision making process. In order to assign the most optimum weight to each criterion, issues such as entropy variation of the OWA weights and the fairness of weight distribution needs to be closely examined for future research endeavour. While the concept of ‘ordered weight’ is being treated as the most important element for the OWA method, this concept cannot be easily translated into Bayesian approach. The idea of reasonableness, which is usually related to the OWA weight assignment strategy, is also applicable to Bayesian fusion. Unfortunately, this area of research has not been widely explored by the fusion community. Assuming conflicting likelihoods are received by the sensors, one proposed method is to re-evaluate the expected cost criterion for the hypotheses that are not mutually exclusive [14], but this method is only limited to Bayesian based fusion. The idea of hypotheses cannot be easily mapped to the linguistic based multi-criteria information, and therefore it is difficult to identify the compatibility between any two measures in the heterogeneous fusion. As such, more works are required in this area, in order to examine a robust methodology to resolve conflicting information in the heterogeneous fusion framework. Finally, future research is also required to examine the usefulness of HOI reports, when it is not provided in a timely manner. Despite the promising results shown in this paper, performance of heterogeneous fusion does not gain any significant improvement by receiving the in-frequent/low-quality HOI reports, yet they are still required to carry the additional computational burden. Having said that, a good quality report could introduce a positive impact to the overall fusion, however, it is difficult to quantify the integrity of this HOI information. This issue raises the concern to the role of conflicting information which is commonly seen in most fusion systems. Our follow-up work will attempt to address the issues identified in this area.

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